Turbulent Liquid-Sodium Flow Induces Magnetic Dipole in a Laboratory Analogue of the Geodynamo

Supercomputers still can't simulate the self-excitation of planetary dynamos. So experimenters have taken up the challenge.

As evidence for a liquid-metal Earth core was beginning to accumulate early in the last century, Joseph Larmor suggested in 1919 that dynamo action of that conducting fluid circulating in Earth's interior might be what sustains the geomagnetic field. But Larmor's idea, which geophysicists now take for granted, lay dormant for the next two decades, even as seismological evidence for a liquid core surrounding a solid iron inner core became ever more detailed.

Why such indifference to a plausible answer to one of nature's great puzzles? "A self-exciting natural dynamo was, at the time, widely thought to violate Lenz's law," explains Johns Hopkins geophysicist Peter Olson. Also off-putting was Thomas Cowling's 1933 annunciation of the first of several "antidynamo theorems." Starting with the Maxwell equations, Cowling proved that perfectly axisymmetric flow of a conducting fluid cannot generate and sustain an axisymmetric magnetic field. Some physicists concluded, therefore, that planetary and stellar magnetic fields were evidence of an entirely new term in the Maxwell equations that manifests itself only in big rotating bodies.

Modern geodynamo theory began in the 1940s when Walter Elsasser ruled out such an exotic addendum to the Maxwell equations and also putative thermoelectric effects. He developed a formalism for applying magnetohydrodynamics (MHD) to the convective motion of the liquid core in the hope of demonstrating the self-excitation of the geomagnetic field from an insignificant seed field. But the equations of MHD, which combine the Maxwell equations with the laws of fluid flow, are far too complex to yield straightforward analytic solutions that demonstrate the geodynamo. Cowling's theorem tells us that the planetary dynamo problem is intrinsically three-dimensional. It requires number crunchers.

But even with today's supercomputers, the vastly different scales of magnetic and hydrodynamic phenomena in Earth's liquid core make realistic numerical simulation impossible. Magnetic structures in the core have typical sizes of a few hundred kilometers. But hydrodynamic turbulence, which is thought to be essential to the geodynamo's operation, is important on scales of order 10 meters. So a realistic simulation that could demonstrate the validity of standard geodynamo theory would require something like 10^{15} grid points.

Experiments with liquid sodium

That's where laboratory model simulations come in. In recent years, almost a dozen groups around the world have been investigating aspects of geodynamo theory using liquid metal circulating in a variety of experimental configurations that attempt to simulate features of Earth's liquid core.[1] The most recent report of results comes from the Madison Dynamo Experiment, an undertaking at the University of Wisconsin headed by Cary Forest.[2] The group used a spherical 1-meter-diameter vessel filled with liquid sodium (see figure 1 and the cover of this issue) to address the role of MHD turbulence in generating and maintaining the axial dipole component that dominates Earth's external magnetic field.

For such purposes, liquid sodium is the experimenter's surrogate of choice. It has about the same low viscosity as liquid iron. (Neither is significantly more viscous than water.) But it's a better conductor, and very conveniently, sodium melts at a much lower temperature (98 °C). It does react violently with water, but the use of liquid sodium as a coolant in nuclear reactors has produced an extensive lore on its safe handling.

Some of the experimental groups spin their vessels to simulate Earth's rotation. But the Madison sphere is stationary. Its axis is defined by the drive shafts of two propellers, one in each hemisphere, counterrotating in the liquid at adjustable rates up to 1300 rpm. In the experiment just reported,[2] the nonmagnetic vessel sat in an almost uniform axial magnetic field $\mathbf{B}_0$ produced by an external pair of large Helmholtz coils. Arrays of Hall probes inside and outside the vessel measured any additional field induced by the propeller-driven flow of the sodium.

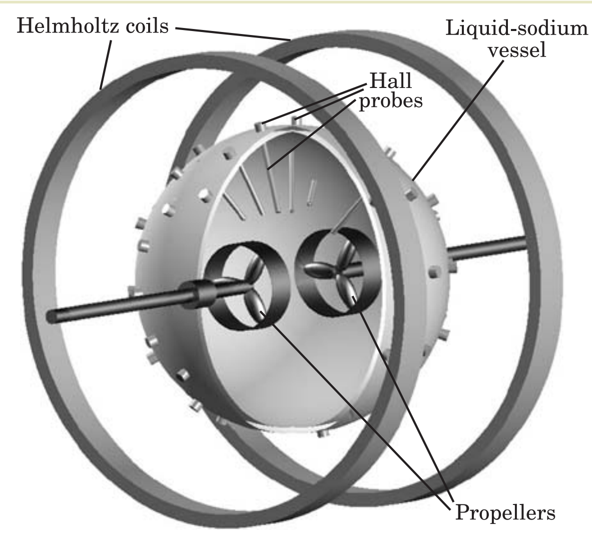

Figure 1. In the Madison Dynamo Experiment, liquid sodium filling a 1-meter-diameter spherical vessel is agitated by two counterrotating propellers. Helmholtz coils, coaxial with the propeller shafts, impose an almost uniform axial magnetic field on the vessel. Arrays of Hall probes measure the resultant magnetic field inside the liquid and on the vessel's surface. (Adapted from ref. 2.)

To confront the constraints imposed on planetary dynamos by Cowling's theorem, the Wisconsin group designed its vessel and propellers to assure that the mean large-scale flow of the liquid sodium is axisymmetric. Therefore, if the experiment did induce an axial dipole moment, one could attribute it to symmetry-breaking departures from the large-scale mean flow.

Strictly speaking, Cowling's theorem refers only to self-excitation from a negligible seed field. But the Wisconsin group's B_0 was far from negligible. To help things along in the experiment, B_0 was set as high as 100 gauss (0.01 T). So Forest and company had to prove a corollary to the effect that axisymmetric flow cannot induce an axial dipole moment even in the presence of a significant preexisting axial field.[2]

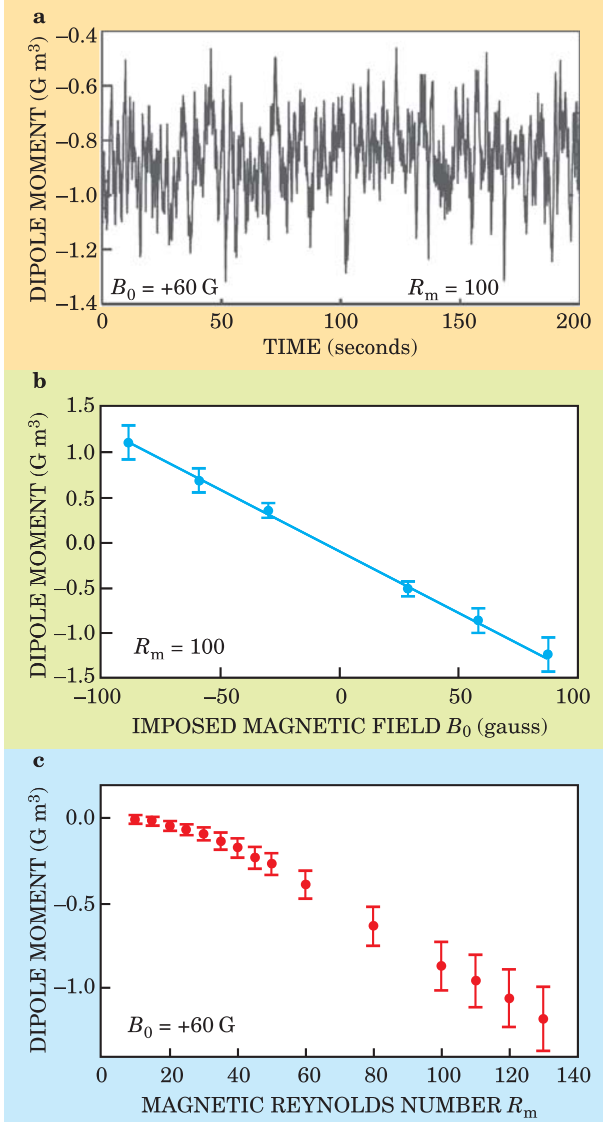

Figure 2. Axial magnetic dipole moment induced in the Madison Dynamo Experiment. **(a)** Dipole moment sampled for several minutes at 1 kHz with imposed axial field B_0 = +60 G and propellers rotating at 1000 rpm, corresponding to magnetic Reynolds number R_m = 100. The time-averaged induced dipole moment is clearly nonzero. **(b)** Time-averaged dipole moment as a function of B_0 at R_m = 100, shown with linear fit. The imposed field tends to induce a dipole moment of opposite sign. **(c)** Variation of time-averaged dipole moment with magnetic Reynolds number at B_0 = +60 G. R_m is taken to be 1/10 the propeller rate in rpm. (Adapted from ref. 2.)

Breaking axial symmetry

Following the pioneering work of Elsasser's protégé Eugene Parker in the 1950s, theorists nowadays usually model the creation of a planetary dynamo as a two-component process. The first component presents no great puzzle in an axisymmetric rotating system. The highly conductive core fluid stretches weak pre-existing magnetic flux in the azimuthal direction, thus creating a toroidal flux field (parallel to latitude lines) in the planet's interior.

What's required, however, for an external dipole field like Earth's is toroidal *electric current* circulating in the core. And Cowling's theorem asserts that perfectly axisymmetric flow of a neutral conducting fluid cannot generate such currents. Parker suggested that the requisite symmetry-breaking flows are provided by cyclonic turbulence in convective transport of iron between the hotter inner region of the 2300-km-thick liquid core and its cooler outer reaches.

In the 1960s, Max Steenbeck and colleagues introduced a quasilinear approximation to elaborate Parker's idea into what is now called the turbulent α effect: Convective and Coriolis forces produce turbulent helical flow that generates a toroidal electromotive force presumably sufficient to drive the currents that maintain the geomagnetic field's dipole moment.

Liquid-sodium experiments at the University of Latvia in Riga[3] and the Karlsruhe Research Center in Germany[4] have, in fact, already used helical flow to produce self-excited dynamo action without a boost from any imposed magnetic field. But in those experiments, the helical flow was imposed by piping and baffles that have little relevance to the much less constraining geometry of Earth's liquid core.

The ultimate goal of all the experimental groups is to demonstrate a self-excited and sustained dynamo in a more Earthlike unconstrained geometry. That goal is at least a few years off. Its achievement should shed light on important geodynamo problems. Why, for example, does the magnetic field at Earth's surface field saturate at about 0.5 G?

"Meanwhile, we've been looking for the first clear laboratory evidence of the turbulent α effect in an unconstrained geometry," says Forest. MHD dynamo action scales like $L_0 v_0$, the product of the system's linear size and its typical fluid velocity. In laboratory experiments, thermal convection cannot create velocities high enough to compensate for the small apparatus size. Therefore the experimental groups drive the liquid metal mechanically. In the Wisconsin experiment, it's done by the propellers.

For different values of B_0 and propeller speed, Forest and company measured the resultant magnetic field at various points inside the vessel and on its surface. Because the field fluctuated wildly in the agitated fluid, the Hall probes were interrogated at 1 kHz for several minutes at each setting. To search for an induced axial dipole moment, the experimenters subtracted the pre-existing $\mathbf{B}_0$ from the measured field configuration. Figure 2a shows that the induced dipole moment at B_0 = 60 G, for all its fluctuation, clearly has a nonzero average.

Figure 2b shows how the mean induced dipole moment varied with B_0. Curiously, the sign of the dipole moment turned out to be systematically opposite to that of B_0. Whether the turbulent α effect can produce such sign reversal is not yet clear. But there is a possible alternative that could perhaps explain it: The so-called turbulent γ effect might be generating a toroidal emf, not by inducing helical flow but by preferentially expelling magnetic field from local regions of higher-than-average turbulence. And the γ effect is explicitly diamagnetic.

"We won't know precisely what turbulent mechanism is inducing the dipole moment until we can map the flow and the internal magnetic field in finer detail," says Forest. "That's our next priority." But first he and his coworkers had to convince themselves that the dipole moment was really induced by turbulence rather than by some symmetry-breaking large-scale flow inadvertently caused by incidental plumbing details. To that end they noted the absence of the higher-order magnetic moments one would expect if the mean flow were sufficiently asymmetric. And then, just to be sure, they rearranged some of the hardware.

Magnetic Reynolds number

The creation and maintenance of an MHD dynamo is a competition between the buildup (advection) of magnetic field by Faraday induction and its diffusion by ohmic dissipation. The ratio of advection to diffusion in a particular system is characterized by the dimensionless magnetic Reynolds number R_m, given by $L_0 v_0 \sigma\mu$, where σ and μ are the conductivity and magnetic permeability of the conducting fluid. Numerical simulations have put the minimum R_m required for a self-excited laboratory dynamo somewhere between 100 and 1000. "Five years ago, we still thought the critical R_m was less than 100," recalls UCLA theorist Steven Cowley. "But new computer simulations suggest that small-scale turbulence pushes that threshold higher.[5] We need experimental input to tell us just how high."

Figure 2c shows the Wisconsin experiment's induced dipole moment (with B_0 fixed at +60 G) for different R_m. The group takes L_0 to be 0.5 m and v_0 to be the tip velocity of the 15-cm-long propeller blades. For each data point, the propellers' rotation rate (in rpm) is approximately 10 R_m.

Creating a self-excited dynamo requires more than just an adequate R_m. The linearity of the dipole moment's dependence on B_0 in figure 2b indicates that the back-reaction of the Lorentz force on the mean fluid flow is negligible in the Wisconsin experiment's weak magnetic field. For sustained dynamo action, however, a system must be in the so-called MHD regime, where the Lorentz back-reaction becomes important. "To see significant back-reaction, we'll eventually have to crank B_0 up to 200 G," says Forest.

Daniel Lathrop and coworkers at the University of Maryland have already carried out experiments in the MHD regime, albeit at lower R_m. In such an experiment,[6] the Maryland group has observed the onset of back-reaction patterns resembling the magnetorotational instability effect predicted in 1959 by Evgeny Velikhov. Those instabilities, clearly important for astrophysical accretion disks, may also play a role in planetary dynamos.

Lathrop and company are in the process of building an experimental apparatus whose unprecedented size holds out the prospect of becoming the first self-excited laboratory dynamo with unconstrained Earthlike geometry. Its 3-meter-diameter rotating sphere will contain about 27 times as much liquid sodium as the Wisconsin experiment. "We'll have an R_m of about 900, similar to Earth's," says Lathrop. Olson jests that the Maryland group "could make headlines with either a self-excited dynamo or a spectacular sodium spill."

Bertram Schwarzschild

References

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Experiment Tracks the Progress of a Chaotically Mixed Chemical Reaction

Imaging an electromagnetically stirred tray of chemicals under diverse conditions reveals surprisingly uniform behavior.

If you drop a spoonful of sour cream into a bowl of borscht, the two liquids will barely mix. But if you stir them, the spoon will drag filaments of cream through the soup. Stir further, and the filaments will stretch and fold. Eventually, if that's your taste, the cream and soup will blend.

How stirring converts a spatially inhomogeneous state into a homogeneous one seems like the sort of problem G. I. Taylor might have solved in the 1930s. But only in the past 20 years have the mathematical tools become available to build plausible theories.

Those theories have advanced to include components that react with each other. Now, they're being tested in the lab. Paulo Arratia of the University of Pennsylvania in Philadelphia and Jerry Gollub of Haverford College near Philadelphia have developed an experiment that tracks the progress of a chemical reaction in a stirred solution.[1]

To their surprise, Arratia and Gollub discovered they could predict the growth of chemical product under a variety of conditions by measuring a single parameter of the fluid flow. The parameter, the mean Lyapunov exponent, characterizes the rate at which fluid elements stretch and separate.

Arratia and Gollub's experiment runs in a time-periodic regime called chaotic advection. Like turbulence, chaotic advection separates fluid elements exponentially in time, but the flow is less vigorous. Chemical engineers, who want to mix things efficiently, prefer turbulence, but for viscous liquids or small vessels, chaotic advection is sometimes the only choice.

Stirred not shaken

Flows stretch and move fluid elements, thereby increasing the opportunities for the reactants to meet and combine. To measure the stretching, Arratia and Gollub used an apparatus that Gollub developed in 2002 with Haverford's Greg Voth and MIT's George Haller.

In the two-dimensional setup, the fluid occupies a shallow square tray, 15 cm wide and 5 mm deep. The fluid's velocity field is measured by imaging the changing positions of 500 microscopic latex tracers dispersed in the fluid.

To create a velocity field in the first place, one needs to shake the container or stir the fluid. Both methods create troublesome inhomogeneities at the walls of the container or at the edges of the stirrer. Voth, Haller, and Gollub solved the boundary problem by using an electromagnetic technique pioneered in the 1990s by Patrick Tabeling of the School of Industrial Physics and Chemistry in Paris.

They make the fluid electrically conductive and position the tray on top of an array of 30 or so magnets. Applying a periodic electric field across the tray drives the fluid back and forth over the magnets. Lorentz forces do the stirring. Arranging the magnets in a regular or random pattern creates a more or less symmetric